Upon adding a capacitive interstage coupling mechanism, appreciable bridging coupling, K_{14}, was realized, and the general filter response curve of Fig. 3 was obtained. Pass band insertion loss (primarily dissipation) was 1.0 dB and frequencies of peak rejection were realized on both filter skirts. It should be emphasized that realization of this performance depends upon using magnetic couplings (K_{12} and K_{23}) and electric coupling (K_{14}) that are in phase opposition. This type of coupling technique will not work for interdigital filter structures in which magnetic and electric couplings are in phase.

Measured data on the various filter couplings is tabulated below for different center frequencies f_0:[4]

Coupling	f_0 = 1900 Mc/s	f_0 = 2000 Mc/s	f_0 = 2100 Mc/s
Δf_{12}	24.8 Mc/s	25.0 Mc/s	26.0 Mc/s
Δf_{23}	14.4 Mc/s	15.5 Mc/s	16.1 Mc/s
Δf_{14}	2.8 Mc/s	3.3 Mc/s	4.0 Mc/s

The absolute coefficient of coupling between the ith and jth resonators will be $K_{ij} \cong \Delta f_{ij}/f_0$, where f_0 is the center frequency and Δf_{ij} is a coupling bandwidth. It can be seen that Δf_{12} and Δf_{23} are relatively insensitive to changes in center frequency, while Δf_{14} is quite frequency sensitive.

The normalized frequencies of peak rejection can be determined from (1):

$$x_p = \pm \sqrt{k_{23}^2 + k_{12}^2 k_{23}/k_{14}}, \qquad (1)$$

where

$x_p = \dfrac{2(f_p - f_0)}{\Delta f_{3\,\mathrm{dB}}}$ normalized frequency of peak rejection

f_p = frequency of peak rejection, and

$\Delta f_{3\,\mathrm{dB}}$ = relative 3-dB bandwidth of general filter when $K_{14} = 0$.

Let

$$k_{12} = \frac{\Delta f_{12}}{\Delta f_{3\,\mathrm{dB}}} = \frac{25}{29} = 0.862$$

$$k_{23} = \frac{\Delta f_{23}}{\Delta f_{3\,\mathrm{dB}}} = \frac{15.5}{29} = 0.534$$

$$k_{14} = \frac{\Delta f_{14}}{\Delta f_{3\,\mathrm{dB}}} = \frac{3.3}{29} = 0.114.$$

Using (1), $x_p = \pm 1.94$. From Fig. 3, peak rejection has been obtained at frequencies of 2026 Mc/s and 1969.6 Mc/s. These actual frequencies correspond to normalized frequencies of $x_p = +1.79$ and $x_p = -2.10$. The average value of $|x_p|$ is 1.945. This checks quite closely with the theoretical value of $|x_p| = 1.94$. The asymmetrical locations of the actual frequencies of peak rejection can be primarily attributed to the frequency sensitivity of k_{14}.

The normalized valley frequencies can be determined from (2):

$$x_v = \pm \sqrt{2}\, x_p \qquad (2)$$

where

x_v = normalized valley frequency.

Using (2) and $x_p = \pm 1.94$, $x_v = \pm 2.75$. From Fig. 3, valleys have occurred at actual frequencies of 2041.5 Mc/s and 1959.5 Mc/s. These correspond to normalized frequencies of $x_v = +2.86$ and $x_v = -2.79$.

The peak rejection attainable can be determined from (3):

$$\text{I.L.} \cong 80 \log |x_p| + 20 \log \left[\frac{k_{12}^2 k_{23}}{2k_{14} d_2 |x_p|}\right] \text{dB} \qquad (3)$$

where

$d_2 = \dfrac{Q_T}{Q_{UL}}$ = normalized dissipation factor of second and third resonators

$Q_T = \dfrac{f_0}{\Delta f_{3\,\mathrm{dB}}}$ = total Q of filter, and

Q_{UL} = unloaded Q of second and third resonators.

Letting $\Delta f_{3\,\mathrm{dB}} = 29$ Mc/s and $f_0 = 2000$ Mc/s, $Q_T = 69$. For the levels of pass band dissipation losses encountered in this filter structure, Q_{UL} is estimated to be about 2300. Then $d_2 = 69/2300 = 0.03$. For $|x_p| = 1.94$, $k_{12} = 0.862$, $k_{23} = 0.534$, and $k_{14} = 0.114$, the theoretical peak rejection from (3) is 52.5 dB. This checks quite closely with measured values of peak rejection.

The insertion loss at the valley frequencies can be determined from (4):

$$\text{I.L.} \cong 80 \log |x_v| + 20 \log \left[\frac{k_{12}^2 k_{23}}{k_{14}(x_p^2 - x_v^2)}\right] \text{dB}. \qquad (4)$$

Letting $|x_v| = 1.75$ and using (4), a theoretical valley insertion loss of 40 dB is obtained. This can be compared to measured values of 39.2 dB and 38.9 dB.

Reasonably good correlation has been obtained between theoretical and measured performance. The general microwave filter structure discussed herein provides frequencies of peak rejection on both filter skirts without using techniques such as m-derived filter circuits or cascading band-pass and band-reject filters. The addition of the bridging coupling, K_{14}, can be implemented without modifying the overall filter form factor or substantially increasing the filter weight or fabrication cost.

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Band-Pass Spurline Resonators

The spurline resonator, a term coined by Schiffman [1] to denote the particular type of transmission-line resonator shown in Fig. 1, is sometimes useful in the design and construction of transmission-line band-stop filters. Its open-wire-line equivalent (see Fig. 1) shows that it is naturally applicable to band-stop (or low-pass) transmission-line filters. A similar structure, which might be denoted a "band-pass spurline resonator" (to distinguish it from the band-stop type of spurline resonator), is shown in Fig. 2. Note that the main difference in the physical configuration of Fig. 2 from that of Fig. 1 is the grounding of the spurline. The main differences in the open-wire-line equivalent circuit is that the shunt stub is short-circuited at its end rather than open-circuited, and a transformer is required at Port 2. The band-pass spurline resonator may provide for alternative realizations of band-pass filters that have, either totally or in part, open-wire-line equivalent circuits of short-circuited shunt stubs alternating with commensurate length transmission lines. Examples are the parallel-coupled-resonator band-pass filter [3], [4] and the direct-coupled-stub band-pass filter [5], [6]. The transformer may, perhaps, be utilized to beneficially alter the impedance level within the filter. It is expected that the band-pass spurline resonator configuration will be most useful in filters of moderate bandwidth.

Design equations for spurline resonators are given in the following. Those for band-pass spurline resonators are believed to be new, while those for band-stop spurline resonators are an alternative form to equations originally derived by Schiffman for asymmetrical bandstop spurlines [7]. Define

$$A = \frac{Y_{oe}^a + Y_{oo}^a}{2} \qquad (1)$$

$$B = \frac{Y_{oe}^b + Y_{oo}^b}{2} \qquad (2)$$

$$D = \frac{Y_{oe}^a - Y_{oo}^a}{2} = \frac{Y_{oo}^b - Y_{oe}^b}{2}, \qquad (3)$$

where Y_{oo}^a, Y_{oe}^a are the odd- and even-mode admittances [2], respectively, of line a; and Y_{oo}^b, Y_{oe}^b are similarly defined for line b.

The inverse relationships for (1), (2), and (3) are

$$Y_{oe}^a = A - D \qquad (4)$$

$$Y_{oo}^a = A + D \qquad (5)$$

$$Y_{oe}^b = B - D \qquad (6)$$

$$Y_{oo}^b = B + D. \qquad (7)$$

Define also

$$k^2 = \frac{D^2}{AB}, \qquad (8)$$

where, by virtue of (1), (2), and (3),

$$0 \leq k^2 \leq 1. \qquad (9)$$

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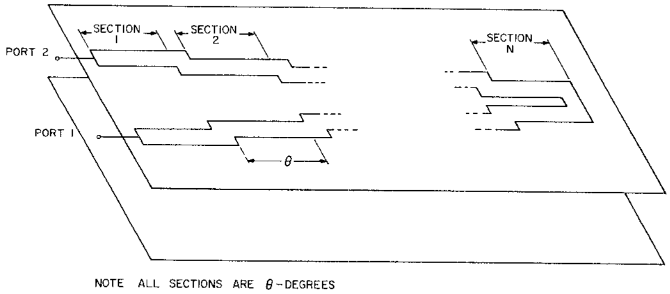

Fig. 1. Band-stop spurline resonator and its open-wire-line equivalent network.

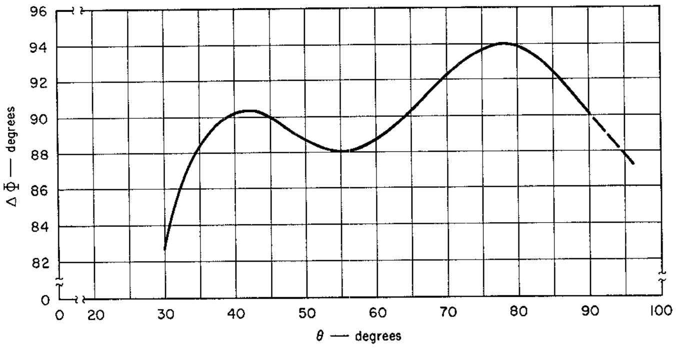

Fig. 2. Band-pass spurline resonator and its open-wire-line equivalent network.

It has been shown [8] that the quantity k^2 is the power-coupling coefficient of lines a and b had these lines been designed to be a nonsymmetrical [8] or conventional directional coupler. Hence, this parameter is useful in aiding the designer to visualize the degree of "coupling" between the two lines.

Design Equations for Band-Pass Spurline Resonators

For physical realizability, choose

$$k^2 < (1 + Y_{12}/Y_1)^{-1}. \tag{10}$$

Then,

$$A = \left(\frac{k}{\sqrt{1-k^2}}\sqrt{Y_1} + \sqrt{Y_{12}}\right)^2 \tag{11}$$

$$B = \frac{Y_1}{1-k^2} \tag{12}$$

$$D = k\sqrt{AB}. \tag{13}$$

The inverse relationships are

$$Y_1 = \frac{AB - D^2}{A} \tag{14}$$

$$Y_{12} = \frac{(A-D)^2}{A} \tag{15}$$

$$N = \frac{A}{A-D}\cdot \tag{16}$$

For symmetrical lines (i.e., $A = B$), Y_1 must be greater than Y_{12} and

$$k = \frac{Y_1 - Y_{12}}{Y_1 + Y_{12}}\cdot \tag{17}$$

An alternative set of equations for band-pass spurline resonators in which N, the transformer turns ratio, is an independent variable is

$$A = Y_{12}N^2 \tag{18}$$

$$B = Y_1 + Y_{12}(N-1)^2 \tag{19}$$

$$D = Y_{12}N(N-1). \tag{20}$$

Here, for physical realizability, N must satisfy

$$1 \leq N \leq 1 + Y_1/Y_{12}. \tag{21}$$

Design Equations for Band-Stop Spurline Resonators

For physical realizability, choose

$$k^2 < (1 + Y_1/Y_{12})^{-1}. \tag{22}$$

Then,

$$A = \frac{Y_{12}}{1-k^2} \tag{23}$$

$$B = \left(\frac{k}{\sqrt{1-k^2}}\sqrt{Y_{12}} + \sqrt{Y_1}\right)^2 \tag{24}$$

$$D = k\sqrt{AB}. \tag{25}$$

The inverse relationships are

$$Y_1 = \frac{(B-D)^2}{B} \tag{26}$$

$$Y_{12} = \frac{(AB - D^2)}{B}\cdot \tag{27}$$

For symmetrical lines (i.e., $A = B$), Y_{12} must be greater then Y_1 and

$$k = \frac{Y_{12} - Y_1}{Y_{12} + Y_1}\cdot \tag{28}$$

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Two Nomograms for Coupled-Line Sections for Band-Stop Filters

A common form of microwave band-stop filter consists of open-circuit shunt stubs separated by lengths of transmission line, all one quarter wave long at band center. Two equivalent but more compact types derivable from the former are the spurline and the parallel-coupled resonator form of band-stop filter.[1] Since the process of designing a filter starting with the prototype and ending with practical physical dimensions can be time consuming and tedious, two nomograms are here presented to speed one step in that process, viz., conversion of the stub type filter into one containing either spurline or coupled-resonator sections (or both).

Equations for the spurline resonator are given in Fig. 1 and those for the coupled-line resonator in Fig. 2. In each case the distributed capacitances between neighboring lines normalized to that of the medium are given in terms of the characteristic admittances of the equivalent stub and connecting line. The method of using the nomograms, Figs. 3 and 4, is explained in the small skeleton nomograms in the lower left corners of those figures. The nomograms have been designed for an impedance level $Z_0 = 1/Y_0 = 50$ ohms, and for $\epsilon_r = 1$ (air dielectric); however, other values may be used by applying multiplicative factors. For example, where an impedance level $Z_0' \neq 50$ ohms is desired, multiply all values of C/ϵ determined in the above manner by $(50/Z_0')$, and where a relative dielectric constant $\epsilon_r \neq 1$ is to be used, divide all values C/ϵ by $\sqrt{\epsilon_r}$. Once the final values of C/ϵ are determined, they may be translated into physical dimensions of round[2] or rectangular[3] coupled striplines from published graphs. The range of the

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